8: Spherical coordinates

Spherical coordinates use the distance $\rho$ to the origin as well as two angles $\theta$ and $\phi$. The first angle $\theta$ is the polar angle in polar coordinates of the $xy$ coordinates and $\phi$ is the angle between the vector $\vec{OP}$ and the $z$-axis. The relation is

$$(x, y, z) = (\rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi))$$.

There are two important figures to see the connection. The distance to the $z$-axis $r = \rho \sin(\phi)$ and the height $z = \rho \cos(\phi)$ can be read off by the left picture the $rz$-plane, the coordinates $x = r \cos(\theta), y = r \sin(\theta)$ can be seen in the right picture the $xy$-plane.

Here are some surfaces described in spherical coordinates.

1. $\rho = 1$ is a sphere.
2. The surface $\phi = 3\pi/4$ is a single cone.
3 The surface $\phi = \pi/2$ is the $xy$-plane.

4 The surface $\sin(\theta) = \cos(\theta)$ is a **plane** if we take the liberty to allow on the $z$-axes any $\theta$ value.

5 $\rho = \phi$ is an **apple shaped surface**. We plot only half of it

6 $\rho = 2 + \cos(13\theta)\sin(\phi^2) + \cos[s]$ is an example of a **jelly fish** The radius $\rho$ depends on the two angles. Jelly fish are cool as they do not die!

7 Match the two surfaces below with either

$$\rho = |\sin(3\phi)|$$

and

$$\rho = |\sin(3\theta)|$$

in spherical coordinates $(\rho, \theta, \phi)$.