What is $6 \div 2(2 + 1)$?

Battles about mathematical Syntax

Oliver Knill, Stem club, April 28, 2018
Plan for today

7 short units

A. Battles
B. Experiments
C. History
D. More riddles
E. Social Media
F. Literature
G. Conclusions
A) Battles
What is $\frac{2x}{3y} - 1$ for $x = 9$ and $y = 2$?
What did you get?
\[ x = 9; \]
\[ y = 2; \]
\[ 2x / 3y - 1 \]
11
Where did it come from?
"The problem $\frac{2x}{3y}-1$ with $x=9$ and $y=2$ was actually posed for a 5th grade homework question and it is interesting how much debate has occurred between our parents. The students who had help from parents had the answer of 2. The teacher explained that the correct answer was 11."
Please Excuse My

Website entry

x=9

Dear Aunt Sally

2x/3y-1

y=2
Fall 2017

\[ \frac{x^3}{3} + y = \frac{x}{3} + y \]

PEMDAS:
- Please
- Exponents
- Multiply
- Divide
- Add
- Subtract
You were called an idiot so I thought it fair you should be informed to defend yourself...

Reference this link for the debate: https://www.youtube.com/watch?v=URcUvFlU1hQ

In the silly debate about a YouTube video on the equation 6+2(1+2), poster Lennard Church disputed your article http://www.math.harvard.edu/~knill/pedagogy/ambiguity/

Timothy, whoever wrote that article is an idiot.
Notation
Music and Math

Kepler: 1571-1630

Kepler: musical notation

Music Notation
Prelude
Op. 28, No. 7

Frederic Chopin
$$(1''') \quad \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial z} \right)^2 - \frac{2m}{K^2} \left( E + \frac{e^2}{r} \right) \psi^2 = 0 .$$

$$r = \sqrt{x^2 + y^2 + z^2} .$$

Und unser Variationsproblem lautet

$$(3) \quad \delta J = \delta \int \int \int dx \, dy \, dz \left[ \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 + \left( \frac{\partial \psi}{\partial z} \right)^2 - \frac{2m}{K^2} \left( E + \frac{e^2}{r} \right) \psi^2 \right] = 0 ,$$

das Integral erstreckt über den ganzen Raum. Man findet daraus in gewohnter Weise

$$(4) \quad \frac{1}{2} \delta J = \int df \, \delta \psi \frac{\partial \psi}{\partial a} - \int \int \int dx \, dy \, dz \, \delta \psi \left[ \Delta \psi + \frac{2m}{K^2} \left( E + \frac{e^2}{r} \right) \psi \right] = 0 .$$

Es muß also erstens

$$(5) \quad \Delta \psi + \frac{2m}{K^2} \left( E + \frac{e^2}{r} \right) \psi = 0 .$$

1) Es entgeht mir nicht, daß diese Formulierung nicht ganz eindeutig ist.
B) Experiments
What is $6 ÷ 2(2+1)$?

$(6/2) (2+1) = 3*3 = 9$  (PEDMAS)

$6/(2*3)= 1$  (PEMDAS)
What is $4x$ for $x=3$?

43 (decimal writing) or 12 (3x4)?

What is $4\frac{1}{2}$?

2 (4 times 1/2) or 4.5?
Is $2 < 2$ true?

especially in colloquial language, we mean with "less" often also "less or equal".
Is \( 0 \in \mathbb{N} \)?

This is not uniform. Some include 0 in the natural numbers, others not. 

ISO 80000-2, Notion of "Whole numbers"
What is \( \log(10) \)?

Most mathematicians and "grown up" computer algebra systems consider \( \log = \ln \), some legacy frame works in calculus or excel understand with \( \log \) the base 10 \( \log \).
What is \(-2^4\) ?

The common understanding is 16. There can be misunderstanding in that one understand \((-2)^4=16\).
Compute $5\cdot3!$

Is it $15!$ or $5 \cdot 6 = 30$. The common understanding is that the factorial is evaluated before everything else.
C) Some History of Notation
Since when do Humans write Math?
Ishango Bone, -20'000 Years

primes 60 cradle of mathematics
Question 1: Since when do we know the number 1?
Story of 1

THE STORY OF 1

PRESENTED BY TERRY JONES
Question 2:
Since when does one write 0?
Quipu

Incas, Andean cultures
$YBC\ 7289$

$0 + \frac{42}{60} + \frac{25}{3600} + \frac{35}{60^3}$

$1 + \frac{24}{60} + \frac{51}{3600} + \frac{10}{60^3}$
Maya Numerals

36 BC
Stela 2, Chiapa de Corzo, Chiapas
Zero

130 AD Ptolemy → 0

600 Brahmagupta (rules)

Sifr (Arabic) → Sunya (Sanskrit)

Fibonacci

1170–1250

Zephyrum

→ Zefiro

Zero
Question 3:

Since when do we write +?
Answer:
Nicholas Oresme
1320–1382
1 + 1/2 + 1/3 + ... = \infty

harmonic series
Question 4:
When was the first arithmetic printed?
First printed arithmetic
1478 at Treviso in Northeastern Italy
A courier travels from Rome to Venice in 7 days. An other courier starts at the same time and travels from Venice to Rome in 9 days. The distance from Venice to Rome is 250 miles. In how many days will the couriers meet?
Solution

The first courier has speed $\frac{250}{9}$

The second courier has speed $\frac{250}{7}$

Their relative speed is the sum. The time is

$$x = \frac{250}{\left(\frac{250}{7} + \frac{250}{9}\right)} = \frac{1}{\left(\frac{1}{7} + \frac{1}{9}\right)} = \frac{63}{16}$$
E primo per quelui da Roma.

\[
\begin{align*}
\frac{112}{7} & \times \frac{250}{1} = \frac{63}{16} \\
250 & \div 16 = 15.625 \\
750 & \div 15 = 50
\end{align*}
\]

Quellui che vien da Roma hauera fatto miglia
Mercantile Arithmetic,
Johannes Widmann  Leipzig

1489
D) More riddles
What is $6/2(2+1)$?
What is \[
\frac{6}{2(2+1)}
\]?
Teacher: What is 11q–q?

Student: 10q

Teacher: you are welcome
Teacher: What is $2k+k$?

Student: 3000.
What is 2/2/2?
What is $1/2x$?

TI–82: $1/(2x)$

TI–83: $(1/2)x$
What is $2^2$?
What is $0^0$?
What is $0/0$?

What is $x/x$ for $x \to 0$?

What is $x^2/x$ for $x \to 0$?
What is $1+2+3+4+\ldots$

Answer: $-1/12$
E) Social Media
PEMDAS

Message #1 Posted by Les Koller on 24 July 2013, 8:09 p.m.

This has probably been discussed here before, but I'd like some input. Without using ANY technology, just your brain and education, what is the value of

\[ 6 ÷ 2(1+2) \]

What is your justification? Does your HP Calculator give the same answer? How about your TI?

Thanks for humoring me here...I've never seen this one before.

---

Re: PEMDAS

Message #2 Posted by Kimberly Thompson on 24 July 2013, 8:23 p.m.,
in response to message #1 by Les Koller

Les

Social media aside, the answer is in the title PEMDAS. Using the convention from my Engineering studies, I get ONE.

Other conventions will possibly yield other results.
I learned my MATH from mathematicians, while attaining an Engineer Degree. I'm very happy with their tutoring.

SlideRule

Edited: 25 July 2013, 7:45 a.m.

Re: PEMDAS

Message #8 Posted by William L. Drylie on 28 July 2013, 3:41 a.m.,
in response to message #3 by Les Koller

Hi Les;

Physicists and Mathematicians do not do it differently. The answer is = 1. Any competent Algebra text with a review of the basic axioms of Arithmetic will tell you to treat the numerator and denominator as if they were enclosed in parentheses. 6 is one expression and 2(1+2) is another expression. The 2(1+2) gets done first as the 1+2 is enclosed and the parentheses are removed by the distributive property. What is left then is 6/6=1. I showed this to a Physics Professor at Duke University who I am friends with and he said "surely you're joking"? I said "no" and he said "if you are going to make me tell you the answer to that, you will buy me a beer after work, and I may consider not having your degree revoked"!!! (laughing) he said "I...you idiot" How shameful!! I actually treated him to a Bratwurst and an 8 dollar German beer at the Bavarian Brat House in Chapel Hill NC. after work and told him the story. The TI's, even with the N-Spire CX CAS, expect you to have a certain level of expertise with entering Algebraic expressions. If you enter (6)/(2*(1+2)) you will get the correct answer of 1. You can see it is also correct as two expressions and it satisfies TI's syntax for Algebraic expressions. My old 48SX shows the answer as 1, written as you stated the original. The hp does the work.

Sincerely, Bill Drylie

Re: PEMDAS

Message #9 Posted by jep2276 on 28 July 2013, 11:23 a.m.,
in response to message #8 by William L. Drylie

Bill,

Why do people think that 2(x+2) is actually 2*(x+2) and not understanding that 2 is the coefficient of (x+2) and therefore a factor that cannot be separated?
Geniuses Only
look close on details
ReShare to see
you're friends

@Jonathan Chaim

$\begin{align*}
\text{Hexagon} + \text{Hexagon} + \text{Hexagon} &= 45 \\
\text{Bananas} + \text{Bananas} + \text{Hexagon} &= 23 \\
\text{Bananas} + \text{Clock} + \text{Clock} &= 10 \\
\text{Clock} + \text{Bananas} + \text{Bananas} \times \text{Hexagon} &= ??
\end{align*}$
6/2(1+2) or 6÷2(1+2) equals? Final Thoughts
You tube: "moral wrong"

"Order of Operations"

1. Parentheses
2. Exponents
3. Multiplication and Division (from left to right)
4. Addition and Subtraction (from left to right)
Can You Solve This?

6 ÷ 2(1+2) =

What is 6÷2(1+2) = ? The Correct
E) Literature
history of mathematical notation
Enlightening Symbols
A Short History of Mathematical Notation and Its Hidden Powers

Joseph Mazur
From "enlightening Symbols"

Superscripts to show the powers or exponents; $x^2$ to indicate squaring of squaring

Our modern multiplication and proportion symbols appear universally

Vowels to represent unknowns and consonants to represent known quantities

An Index Plan for writing exponents: $10^2 = 30 + 20$ would mean $x^2 = 3x + 21$

Exponents of polynomials are beginning to be indexed by numerals

Our equal sign (=) is introduced

Recognition of the value of complex solutions to polynomial equations

Letters $M$ and $D$ for multiplication and division. So $30D \text{ sec} M \text{ ter} \gamma$ would indicate $\frac{M^2 \times \gamma}{D}$

Symbols $\sqrt{}$, $\sqrt[3]{}$, and $\sqrt[4]{}$ for square, cube, and fourth roots

Powers of an unknown are labeled as $R^2$, $R^3$, and $R^4$, and square root as $R^\frac{1}{2}$

Al-Khwarizmi's book on arithmetic includes the Hindu-Arabic numerals, and organized rhetorical algebraic expressions according to the various species of forms

The first use of zero that we know of (a small black dot) as a number

Abbreviations for unknown, powers, and minus symbol

First printed treatise on algebra

Characters that indicate + and − and square root

The Liber abbaci was a comprehensive western source for abacus methods that included Hindu-Arabic numerals

1687 AD
Isaac Newton's
Principia

1676 AD
Gottfried Leibniz's
De Quadratura Arithmetica

1637 AD
René Descartes's
Geometria

1631 AD
William Oughtred's
Clavis Mathematicae

1591 AD
Thomas Harriot

1585 AD
François Viète

1585 AD
Simon Stevin's
De Thiende

1572 AD
Rafael Bombelli's
L'Algebra

1557 AD
Robert Recorde's
Whetstone of Witte

1545 AD
Gerolamo Cardano's
Ars Magna

1544 AD
Scipione de' Castiglione's
Arithmetica Integra

1525 AD
Christoff Rudolph

500 BC
Plato's Academy

300 BC
Euclid's
Elements

302 AD
Diophantus's
Arithmetica

628 AD
Brahmagupta

1202 AD
Fibonacci's
Liber abbaci

1400 AD
Al-Kashi

1478 AD
Fra Luca Pacioli

1484 AD
Nicolas Chuquet
Florian Cajori
1859–1930
242. Order of operations in terms containing both $\div$ and $\times$.—If an arithmetical or algebraical term contains $\div$ and $\times$, there is at present no agreement as to which sign shall be used first. “It is best to avoid such expressions.”³ For instance, if in $24 \div 4 \times 2$ the signs are used as they occur in the order from left to right, the answer is 12; if the sign $\times$ is used first, the answer is 3.

Some authors follow the rule that the multiplications and divisions shall be taken in the order in which they occur.⁴ Other textbook writers direct that multiplications in any order be performed first, then divisions as they occur from left to right.⁵ The term $a \div b \times b$ is interpreted by Fisher and Schwatt⁶ as $(a \div b) \times b$. An English committee⁷ recommends the use of brackets to avoid ambiguity in such cases.
Mathematical Notation
A Guide for Engineers and Scientists
Edward R. Scheinerman
F) Conclusions
Be clear. Especially if no consensus exists.
"Mathematical notation isn't really a problem about mathematics. It's really more a problem in linguistics. It's not about what mathematical notation could conceivably be like; it's about what mathematical notation as it's actually used is actually like—as it's emerged from history and presumably from the constraints of human cognition and so on. And, in fact, I think mathematical notation is a pretty interesting example for the field of linguistics."

"A seemingly modest change of notation may suggest a radical shift in viewpoint. Any new notation may ask new questions"

Barry Mazur, cited in "Enlightening Symbols"
The end