Dear Undergraduate Advisor,

I'm writing to give you some information about the introductory courses that the Mathematics Department will run this academic year, as well as other courses your students may be interested in. A description of the topics covered in each course is included in this packet; please take a look at the topic lists for the courses that you want your concentrators to take. If you don't find a particular topic that you feel should be required, or you'd like to talk more generally about course content, please email me at jjchen@math.harvard.edu.

Here is a brief synopsis of the courses.

First, our introductory courses:

- Math Ma,b is a two semester sequence that integrates calculus with pre-calculus. The Math M sequence covers all of the material in Math 1a, so a student who completes the Math M sequence is ready for Math 1b. Math Ma is offered in the fall, and Math Mb is offered in the spring.

- Math 1a is a fairly standard first-semester calculus course, covering differential calculus and an introduction to integral calculus. It is taught both semesters but in slightly different formats — in the fall, it is taught in sections of 20-30 students; in the spring, it is taught in a single class. We encourage students to take Math 1a in the fall, both because of the smaller section size and because this gives students the option of switching to Math Ma if that seems appropriate.

- Math 1b is second-semester calculus, covering applications of integration, series, and differential equations. It is offered both semesters.

- Math 21a is a fairly standard introduction to multivariable calculus, offered both fall and spring.

- Math 18 is an alternative to Math 21a; it covers selected topics in multivariable calculus from Math 21a with examples principally from economics and the social sciences. Math 18 greatly overlaps with Math 21a but replaces most of the vector calculus covered in 21a with further economic applications (primarily extra work on optimization, including the Envelope Theorem and Kuhn-Tucker conditions, but also higher order approximations and present values). This is a one semester course, offered only in the fall.

- Math 19a is a course on mathematical modeling and differential equations with examples that come from the life sciences. Math 19a will be offered only in the fall.

- Math 21b is a standard linear algebra course with additional applications to differential equations and Fourier series; it is offered both semesters.

- Math 18b/19b is an introduction to linear algebra, probability, and statistics with a focus on computing and applications to the life and social sciences. The course will only be offered in the spring.

- Math 22a,b covers linear algebra and vector calculus with an introduction to proof. The workload and topics are similar to Math 21. In particular, Math 22a covers roughly the same topics as Math 21b, and Math 22b covers roughly the same vector calculus topics as Math 21a. The course is specifically designed to develop proof writing skills and mathematical thinking, and hence it provides excellent preparation for students interested in the mathematics concentration.

  Math 22a is offered in the fall, and Math 22b is offered in the spring.

- Math 23a,b is a moderately rigorous course in linear algebra and multivariable calculus. It is designed for students who are serious about mathematics and interested in being able to prove the theorems that they use, but who are as much concerned about the application of mathematics in fields like physics and economics as about "pure mathematics" for its own sake.

  Alternatively, students can take Math 23a followed by Math 23c; this sequence is designed for students interested in fields like computer science, statistics, data science, and economics. Math 23c omits almost all the material on differential forms and Stokes's theorem from Math 23b, replacing it with material on the mathematical foundations of probability.

- Math 25a,b and Math 55a,b are also advanced alternatives to Math 21a,b. In this regard, Math 25 and Math 55 are designed principally for prospective mathematics concentrators and are meant to be very homework intensive. Both courses offer a rigorous, modern view of multivariable calculus, linear algebra and associated topics. Math 55 differs from Math 25 in requiring a truly sophisticated math background to start. Math 25 only requires placement into Math 21a and a gung-ho attitude toward mathematics. Math 55 requires the consent of the instructor. These courses and Math 23 will be given at different times so students can shop all three courses and down-shift in course number if the original choice proves untenable.

- Math 101 is an introduction to rigorous mathematics, axioms, and proofs via topics such as set theory, symmetry groups, and low dimensional topology. It is not to be taken by students who took Math 23, 25 or 55. It can be taken concurrently with either Math 21a or Math 21b. It will be given both semesters.

There are also several upper-level courses that may be of particular interest to mathematically oriented students in some applied fields:

- Math 110 develops the theory of inner product spaces and applies it to study differential equations; it will be offered in the spring.

- Math 112 is an introductory real analysis course, to be offered in the spring. Math 114 is a second real analysis course, offered in the fall.

- Math 113 is an introductory complex analysis course, offered in the spring.

- Math 115 covers some of the most important analytical tools widely used in engineering and physics, including complex contour integrals, differential equations, and Fourier analysis. It will be offered in the spring.

- Math 116 is a course in real analysis and functional analysis (infinite-dimensional vector spaces) with the mathematics motivated by optimization problems. It is offered in the fall.

- Math 118r is a spring course on dynamical systems; it integrates rigorous theory with numerous applications of genuine scientific interest.

- Math 121 is a proof-based linear algebra course; it will be offered in the fall.

- Math 152 is a course on discrete mathematics that may be of particular interest to computer science concentrators. It will be offered in the spring.

- Math 153 is a mathematical biology course for undergraduates, offered in the fall; there is also a graduate level course on mathematical biology, Math 243, offered in the spring.

- Math 154 is an introduction to probability theory, offered in the spring.
- Math 157 is an interactive introduction to problem solving with an emphasis on subjects with comprehensive applications. It will be offered in the spring.

With all of these math courses in mind, remember that placement into Math 21a,b on the Harvard Placement test allows (in principle) entry into Math 18, 19, 21, 22, 23, 25, 55, or 101. If you have any concern about the qualifications of a particular student for these courses, please have them visit the math department's open placement advising hours in the first two weeks of the semester; a full schedule of those hours will be at http://www.math.harvard.edu/sectioning/

Course topic lists:

Math Ma

Math Ma and Mb is a two-semester sequence that merges calculus and precalculus. The ideas of calculus are discussed from the start, weaving in background material from precalculus as it is needed; this means that the most important ideas and skills are reinforced for an entire year.

Topics Covered

- Modeling with functions
- Combining and altering functions
- Average rates of change
- Linear functions; slope and linear approximation
- The derivative: definition and interpretation
- Computing derivatives: derivatives of power functions, sums and differences, products and quotients
- Limits and continuity
- Exponential functions and their derivatives
- Polynomials
- Optimization
- Inverse functions
- Logarithms and their derivatives

Math Mb

Math Mb is a continuation of Math Ma.

Topics Covered

- The Chain Rule and Applications
  - The Chain Rule
  - Logarithmic Differentiation
  - Implicit Differentiation
  - Related Rates
- Trigonometry
  - Unit Circle Trigonometry: Sine and Cosine
  - Periodic and Sinusoidal Functions
  - Tangent, Arc Length and Angles
  - Right Triangle Trigonometry
  - Inverse Trigonometric Functions
  - Solving Trigonometric Equations
  - Problem Solving with Trigonometry
  - Derivatives of Trigonometric Functions
  - Derivatives of Inverse Trig Functions
  - Problem Solving with Trig and Inverse Trig Derivatives
- Indeterminate Forms and L'Hôpital's Rule, Relative Growth Rates
- Integration
  - Net Change
  - The Definite Integral
  - Definition of the Definite Integral
  - Properties of the Definite Integral, The Area Function
  - The Fundamental Theorem of Calculus, Part 1
  - The Fundamental Theorem of Calculus, Part 2
  - Antiderivatives
  - Slicing to Find Area
  - Integration by Substitution
  - Slicing to Solve Density Problems
- Introduction to Differential Equations
  - Modeling with Differential Equations
  - Analyzing Differential Equations

Math 1a

Math 1a is a fairly standard first-semester calculus course; it covers single-variable differential calculus, introduces integral calculus, and includes a strong focus on limits.

Topics Covered

- Modeling with functions
- Rates of change: average vs. instantaneous
- Definition and interpretation of the derivative
- The derivative as a function, differentiability
- Limits and continuity
- Exponential, logarithmic, trigonometric, and inverse trigonometric functions, and their derivatives
- Differentiation rules (Product Rule, Quotient Rule, Chain Rule, etc.)
- What does f' say about f?
- Using derivatives to graph functions
- Linear approximation
- Maxima and minima of functions
- Absolute extrema
- Optimization
- Implicit differentiation
- Logarithmic differentiation
- Related rates
- Indeterminate forms and L'Hôpital's rule
- Relative growth rates
- Net change
- The definite integral
- The Fundamental Theorem of Calculus
- Integration by substitution

Math 1b

Math 1b is a second semester in single-variable calculus and covers applications of the integral, Taylor approximations and infinite series, and modeling with differential equations. In the first unit, students build on their notions of the definite integral and the fundamental theorem of calculus from Math 1a to develop the general Riemann sum as a flexible and powerful tool; they also meet the improper integral and numerical methods for integration including error bounding. In the series unit students build from linear approximations and low-order Taylor approximations to Taylor series, which motivates their study of convergence and divergence of infinite series. The final third of the class introduces differential equations as a tool for modeling; students learn to solve first-order separable and second-order linear homogeneous differential equations, but the unit focuses on understanding the qualitative behavior of solutions that can be inferred even when a solution can't easily be described; we emphasize population models, including exponential and logistic growth, and systems for modeling for population interactions.

Topics Covered

- Integration
  - Approximating by a general Riemann Sum
  - The Definite Integral
  - Area and Volume using the Definite Integral
  - Volumes of Revolution
  - 3D Density Problems
  - Integration by u-Substitution
  - Integration by Parts
  - Integration by Partial Fractions
  - Numerical Integration (Midpoint, Trapezoid and Simpson's Approximation)
  - Error Bounding for Numerical Integration
  - Convergence and Divergence of Improper Integrals
  - Comparing Two Improper Integrals to Determine Convergence
- Series
  - Linear and Low-Order Taylor Approximation
  - Taylor Polynomials
  - Taylor Series
  - Convergence and Divergence of Infinite Series
  - Comparing Two Series to Determine Convergence
  - Nth Term Test for Convergence
  - Comparing Series to Integrals to Determine Convergence
  - The Harmonic Series and other p-Series
  - Geometric Series and Applications
  - Asymptotic Comparison of Series

- The Alternating Harmonic Series and Convergence of Alternating Series
  - Absolute Convergence
  - The Ratio Test
  - Convergence of Power Series
  - Representing Functions by a Power Series
  - The Taylor Remainder Formula
  - Approximating Series by a Partial Sum and Error Bounding
- Differential Equations
  - Modeling with Ordinary Differential Equations
  - Slope Fields and the Uniqueness and Existence of Solutions
  - Qualitative Analysis of Autonomous First-Order Linear ODEs
  - Solving Separable Differential Equations
  - Compartmental Analysis and Mixing Problems
  - Euler's Formula and Basics of Complex Numbers
  - Solving Second-Order Homogeneous Linear Differential Equations
  - Modeling Population Interactions with Systems of Differential Equations
  - Interpreting the Phase Plane
  - Shapes of Trajectories of Solutions in the Phase Plane
  - Modeling Epidemics and Other Applications of Systems

Math 18

Math 18 is a one-semester multivariable calculus course aimed at students interested in studying economics. Students generalize their understanding of single-variable calculus to functions of two, three or more variables. There is an emphasis on approximation by, and modeling with, second degree polynomials. A section on difference and differential equations contrasts the discrete and continuous approaches to modeling and problem-solving. A short study of some vector geometry informs our detailed study of optimization. We extend Lagrange multipliers using the Envelope Theorem and the widely-used Kuhn-Tucker method.

Math 18 greatly overlaps with Math 21a, but replaces most of the vector calculus of that course with further economic applications (primarily extra work on optimization, including the Envelope Theorem and Kuhn-Tucker conditions, but also higher order approximations and present values).

Topics Covered

1. **Functions Of Several Variables:**

   Functions of two or more variables (especially Cobb-Douglass functions), surfaces via traces and level curves, partial derivatives, elasticities and the economic interpretation of partial derivatives, linear approximation, second degree polynomial approximations, and modeling using quadratics.

2. **Multiple Integrals:**

   Double integrals over rectangles, iterated integrals, double integrals over general regions, applications of double integrals to probability, double integrals in polar coordinates, triple integrals, and triple integrals in cylindrical coordinates.

3. **Difference Equations And Differential Equations:**

   Geometric series, present discounted value, economic modeling with difference equations, present value of a continuous income stream, differential equations, modeling with and solving differential equations, and an introduction to partial differential equations.

4. **Vectors And Geometry:**

   Vectors in two and three dimensions, the dot product, the cross product, lines, planes, projections, and distances.

5. **Multivariable Derivatives:**

   Vector-valued functions (parametrized curves), the chain rule, directional derivatives and the gradient vector, tangent planes, the implicit function theorem and implicit differentiation, constrained optimization, economic interpretations of Lagrange multipliers, the Envelope Theorem, the Extreme Value Theorem, Kuhn-Tucker (optimization on a region defined by inequalities), and an economic application of the Kuhn-Tucker conditions.

Math 19a

One of the primary goals of Math 19a is to facilitate comprehension of highly-quantitative research in the life sciences. To this end, Math 19a creates an interplay between reading journal articles in biology/medicine and introducing the necessary mathematical methodology. The mathematical content includes: Development and analysis of mathematical models, multivariable calculus, matrix-vector algebra, differential/difference equations in one or more variables, and bifurcations in nonlinear dynamical systems. The biological content may include topics such as: Models of infectious disease transmission, biochemical kinetics and Michaelis-Menten type formalism, ion channel kinetics and action potentials, interacting populations (competition, mutualism, predator-prey), biological waves, chemotaxis, pattern formation. Topic selection may vary according to student interests and preferences. Probability and statistics are not emphasized in Math 19a, as those topics are more suited for Math 18b/19b.

Topics Covered

- first-order differential equations (DEs) as models of biological phenomena
- solving separable and linear DEs
- single-population models (logistic and density-dependent growth, Allee effect, insect outbreak model)
- qualitative analysis of first-order DEs: phase line, equilibria, stability and bifurcations
- planar systems of DEs: phase plane, nullclines and equilibria
- Lotka-Volterra type models of mutualism, competition and predator-prey; SIRS infectious disease models; biochemical kinetics models
- matrix-vector algebra, eigenvalues and eigenvectors
- linear, constant-coefficient systems of DEs and the phase plane
- nonlinear systems, linearization and the Jacobian, stability of equilibria, bifurcations
- periodic solutions of systems of DEs; biological oscillators: action potentials, self-oscillations in glycolysis
- heteroclinic and homoclinic trajectories, prelude to biological waves of invasion and pursuit
- discrete-time systems and one-dimensional mappings, applications to heart rhythm analysis
- higher-dimensional mappings, fixed points, stability, and control of cardiac alternans
- partial differential equations in the biosciences: diffusion, advection, and reaction
- separable partial differential equations and Fourier series
- reaction-diffusion systems and travelling wave solutions
- biological waves of invasion, pursuit and evasion; propagation of action potentials in tissue
- chemotaxis and signalling
- miscellaneous topics (time permitting): chaos and global bifurcations in biological systems

Math 18b/19b

Math 18b/19b is a course on linear algebra, probability and statistics, with a focus on computing and applications to the life and social sciences, including economics. (No background in these topics is presumed or required.) The course covers essentially all of the linear algebra that is covered in Math 21b. Meanwhile, the probability and statistics taught here gives a background that is sufficient for various higher level courses in statistics (such as Stat 111). The course teaches the subjects of linear algebra, probability and statistics hand in hand. Students will learn to use MATLAB to visualize and perform calculations on data sets coming from the life and social sciences.

While the course is designed to complement either 18 or 19a, neither is a prerequisite for 18b/19b.

Topics Covered

Linear Algebra:

- Solving linear systems, Gaussian elimination
- Linear transformations
- Image, kernel
- Basis, dimension, coordinates
- Orthogonality, projection, least squares solutions
- Determinant
- Eigenvectors, eigenvalues
- Singular value decomposition, principal component analysis
- Discrete and continuous dynamical systems

Probability and Statistics:

- Axioms of probability
- Conditional probability, Bayes' Theorem
- Mean, standard deviation, variance
- Random variables
- Basic combinatorics
- Discrete and continuous probability functions
- Expected value
- p-values
- Independence, correlation, covariance
- Model testing, maximum likelihood
- Central limit theorem, hypothesis testing
- Markov matrices

Math 21a

Multivariable calculus extends single variable calculus to higher dimensions. Topics include vector analysis, extrema, Lagrange, integration and differentiation in higher dimensions. It is heavy in applications like vector fields, motion, and visualization in two and three dimensions. A computer algebra component allows students to experiment: art creations, sculpting, and building objects for virtual worlds have come up in the past.

Multivariable calculus is a bridge to many other sciences and more advanced mathematics. It provides vocabulary for understanding fundamental processes and phenomena in our daily life such as sound waves and light waves, heat or wind, finance, epidemiology, optimization, and statistics. In addition, it builds visualization skills which are important when, for example, presenting data or visualizing structure. Math 21a also helps to hone problem solving skills and perseverance.

Topics Covered

- coordinates and distance
- vectors and dot product
- cross product and planes
- lines and planes, distances
- functions of 2 and 3 variables, level surfaces and quadrics
- curves, velocity, acceleration
- arc length and curvature
- other coordinates
- parametric surfaces
- functions and continuity
- partial derivatives and the gradient
- partial differential equations
- linear approximation
- chain rule
- implicit differentiation
- tangent spaces
- directional derivative
- maxima, minima, saddle points
- Lagrange multipliers
- Global extremal problems

- double integrals
- polar integration
- surface area
- triple integrals
- spherical integration
- vector fields
- line integrals
- Fundamental Theorem of Line Integrals
- Green's Theorem
- flux integrals
- Stokes' Theorem
- Divergence theorem

Math 21b

Math 21b covers basic linear algebra, with applications to differential equations and Fourier theory. The course also serves as a bridge to more advanced mathematics by introducing students to more abstract notions such as abstract linear spaces and linear transformations. The emphasis is on understanding how fundamental ideas such as bases and inner products can be applied in a variety of settings, such as least squares, Markov chains, and Fourier series.

Topics Covered

- Systems of linear equations; Gauss-Jordan elimination
- Linear transformations
- Matrix products and inverses
- Image and kernel, rank-nullity
- Subspaces, bases and linear independence
- Coordinates
- Orthonormal bases and orthogonal projections
- Gram-Schmidt
- Least squares and data fitting
- Determinants
- Eigenvalues and eigenvectors, diagonalization over $\mathbb{R}$ and over $\mathbb{C}$
- Linear discrete dynamical systems
- Linear and nonlinear continuous dynamical systems
- Symmetric matrices and the Spectral Theorem
- Linear differential equations, homogeneous and inhomogeneous
- Abstract linear spaces, linear transformations, and inner product spaces
- Fourier series and Parseval's Theorem
- Partial differential equations: heat and wave equation

Math 22a

Math 22a covers roughly the same topics as Math 21b but is also designed to develop proof writing skills and mathematical thinking.

Topics Covered

See Math 21b.

Math 22b

22b covers roughly the same vector calculus topics as Math 21a but is also designed to develop proof writing skills and mathematical thinking.

Topics Covered

See Math 21a.

Math 23a

Mathematics 23a is intended for students who are interested both in learning to prove important mathematical results and in applying those results to fields like physics and economics. It assumes only proficiency with the mechanics of calculus and infinite series, since the theory behind calculus is presented in the real analysis part of the course. Students are expected to master 26 classic proofs, but no prior experience with proofs is assumed. Even international students who have not done the full BC Advanced Placement curriculum can succeed in Math 23a. Because Math 23a does applications in addition to fundamentals, it requires a substantial time commitment.

Target audiences:

- Prospective math concentrators will learn to prove 26 important theorems.
- Prospective physics concentrators will learn mathematics that is used in Physics 15 and 16.
- Mathematically-inclined economists will learn material that is required for graduate programs in the field.
- Prospective CS concentrators will learn proof techniques so that they can skip CS 20.

Topics Covered

- Linear algebra, with an introduction to the R scripting language
  - Week 1 - Vectors and matrices; linear transformations
  - Week 2 - Dot and cross products; geometry
  - Week 3 - Row reduction, independence, basis
  - Week 4 - Eigenvectors and eigenvalues
- Real analysis, including proofs of key theorems
  - Week 1 - Number systems; infinite sequences
  - Week 2 - Infinite series, convergence tests, power series
  - Week 3 - Limits and continuity of functions
  - Week 4 - Derivatives, inverse functions, Taylor series
- Analysis and differential calculus in $\mathbb{R}^n$
  - Week 1 - Topology, sequences, limits in $\mathbb{R}^n$, differential equations
  - Week 2 - Limits, continuity, and compactness in $\mathbb{R}^n$; partial and directional derivatives
  - Week 3 - Differentiability, Newton's method, inverse functions
  - Week 4 - Manifolds, critical points, Lagrange multipliers

Math 23b

Math 23b develops all the algebra, analysis and calculus that is necessary for Stokes's theorem, the generalization of the fundamental theorem of calculus. Students who complete Math 23b are prepared for 100-level mathematics courses that require experience with proofs. They are expected not to take courses that assume no prior experience with proofs, such as Math 101, 112, or 121. The material on differential forms is important for physics and many areas of engineering mathematics, but it has little direct application to economics.

Target audiences:

- Prospective math concentrators will learn to prove 26 more theorems.
- Prospective physics concentrators will learn the mathematics that underlies electromagnetic theory, both in terms of differential forms and of classical "vector calculus."
- Mathematically-inclined economists will become expert with epsilons and deltas and learn some topics that are used in optimization theory.

Topics Covered

- Integration in $\mathbb{R}^n$
  - Week 1 - Definition of the Riemann integral; integrable functions
  - Week 2 - Evaluating integrals, Fubini's Theorem
  - Week 3 - Determinants, volumes, linear change of variable
  - Week 4 - Change of variable; polar, cylindirical, and sperical coordinates
- Algebra and analysis
  - Week 1 - Volumes of manifolds in $\mathbb{R}^n$
  - Week 2 - Abstract vector spaces and change of basis
  - Week 3 - Sequences of functions
  - Week 4 - Lebesgue and improper integrals
- Differential forms
  - Week 1 - Algebra of differential forms
  - Week 2 - Orientation of subspaces and manifolds
  - Week 3 - Boundary orientation; exterior derivative
  - Week 4 - Form fields in the language of div, grad, and curl
  - Week 5 - Stokes's theorem and the Poincaré lemma

Math 23c

Math 23c is an alternative to Math 23b designed for students who are interested in the application of mathematics to fields like computer science, statistics, data science, and economics. Math 23c omits almost all the material on differential forms and Stokes's theorem from Math 23b, replacing it with material on the mathematical foundations of probability. Integration is used to compute expectation and variance rather than center of mass and moment of inertia. About half of the topics are also included in Math 23b.

Topics Covered

- Logic, using boolean algebra and finite fields
- Foundations of probability for finite sample spaces
- Probability for countably infinite sample spaces
- The Riemann integral and its generalizations
- Probability for uncountably infinite sample spaces
- Determinants, volume, liinear change of variables
- Calculating integrals by parametrization
- Generating functions and the central limit theorem
- Abstract vector spaces and linear transformations
- Applications of linear algebra
- Sequences of functions
- Vector Fields

Math 25a

The goal of Math 25 is to introduce students to abstract mathematical thought, through the study of linear algebra (in Math 25a) and multivariable calculus (in Math 25b).

Topics Covered

- sets, functions, cardinality, equivalence relations
- fields and vector spaces
- subspaces, bases, dimension,
- linear maps, matrices, rank-nullity
- matrix multiplication, invertibility
- linear systems, row operations, elementary matrices
- determinants
- eigenvalues, characteristic polynomial
- inner products, dual spaces, adjoints
- spectral theorem

Math 25b

Math 25b is a rigorous introduction to multivariable calculus. This is an honors class, and moves fast. A key goal of the course is to gain experience writing proofs.

Topics Covered

- real numbers, least upper bound property, Dedekind cuts
- sequences and limits
- continuity, intermediate and maximum value theorems
- topology (open, closed, interior/exterior/boundary)
- compactness, Heine-Borel
- function spaces, equicontinuity, Arzela-Ascoli, ODE existence
- differentiability, mean value theorem, partial derivatives, Taylor polynomials, second derivative test
- inverse/implicit function theorems, manifolds, Lagrange multipliers
- Riemann/Darboux integral, Riemann-Lebsegue theorem, Fubini, fundamental theorem of calculus
- differential forms and Stokes' theorem

Math 55a

Math 55a and 55b are extremely intensive courses; students should already be comfortable writing and understanding mathematical proofs. They are intended for students who enjoy learning abstract mathematics enough to devote most of their academic time during the semester to it.

Topics Covered

- vector spaces and linear transformations between them
- linear operators, eigenvectors and eigenvalues
- tensor, symmetric, and exterior products
  - key special cases: duality, bilinear forms, and determinants
- inner products and the adjoint over $\mathbb{R}$ and $\mathbb{C}$
- representation theory of finite groups, and the Sylow theorems

Math 55b

Topics Covered

- Metric and topological spaces
  - metric spaces, topological continuity;
  - completeness and completions;
  - compactness;
  - product topology.
- Integration of functions of one variable
  - Normed vector spaces;
  - Riemann integral;
  - L_1 and L_∞.
- Differentiation:
  - differentiability in one variable;
  - fundamental theorem of calculus;
  - differentiability of functions of several variable;
- Introduction to differential geometry
  - inverse and implicit function theorems.
  - existence of solutions for ODEs;
  - differential forms;
  - differentiable manifolds.
- Integration in several variables
  - integration in $\mathbb{R}^n$;
  - change of variables;
  - integration of differential forms;
  - Stokes theorem.
- Introduction to complex analysis
  - holomorphic functions;
  - the Cauchy formula;
  - maximum principle;
  - Taylor expansions.

Math 101

The goal of this course is to introduce the practice of higher mathematics, with minimal prerequisites. Students learn how to communicate mathematics rigorously by writing mathematical proofs, and they also learn abstract mathematical thought by studying different branches of mathematics.

The topics covered vary depending on the instructor, but here is a possible list.

Topics Covered

- Set theory and logic
  - Basic introduction to statements and logic
  - Naive set theory
  - Functions
  - Equivalence relations
  - Countable and uncountable sets
- Group theory
  - Homomorphisms and isomorphisms
  - Cosets and Lagrange's Theorem
  - Normal subgroups, quotient groups
  - Symmetric and dihedral groups
- Analysis / topology
  - Notions of open, closed and compact sets
  - Metric spaces
  - Continuity

Math 110

This course is designed to match the background of students who have taken Math 23 or 25: expertise in linear algebra and near-total ignorance of differential equations. Infinite-dimensional inner-product spaces are introduced at the start of the course, and differential equations are viewed in terms of linear operators that act on spaces of square-integrable functions. The topics covered overlap substantially with a course like Applied Math 105, but the viewpoint is quite different.

Since the approach lends itself naturally only to linear equations, little is said about nonlinear differential equations. Many of the equations that are considered in detail are motivated by physics, since electromagnetic theory and quantum mechanics are linear, but there is no physics prerequisite.

The emphasis on Hilbert space methods is especially appropriate for physics students. Those who are taking Physics 15c, 143, or 153 find the material immediately relevant and useful.

Students are expected to learn the proofs of key results, but these proofs tend to be straightforward and computational. Where techniques of real analysis are need, they are introduced cautiously.

Prerequisites:

Math 23, 25, or 55 is fine. Sophomores have done very well. The course is open to freshmen who are concurrently enrolled in Math 23b, 25b or 55b.

Students who took Math 21 should also have taken a course where they did proofs. Obvious candidates are Math 101, Math 112, Math 116, Math 121, Math 130, Math 152, and Math 154.

Topics Covered

- First-order linear differential equations
- Second-order equations: Wronskians and Abel's formula
- Second order equations: Laplace transforms and Green's functions
- Hilbert spaces, especially L_2, completeness and separability
- Orthogonality and generalized Fourier series
- Eigenvalues and eigenfunctions for differential operators
- Traditional sine-cosine Fourier series; Sturmian form
- Hermitian operators
- Legendre's and Hermite's operators
- Rodrigues formulas, recurrences, and generating functions for orthogonal polynomials
- Wave, heat, and Schroedinger equations: one space variable, one time variable
- Bessel's operator and Bessel functions
- Laplacian operator in two and three dimensions

Math 112

Math 112 is an introduction to real analysis, giving a rigorous development of calculus, from the construction of the real number system, to the Riemann integral. This course covers foundational material which is of fundamental importance in virtually all advanced mathematics classes.

Topics Covered

- The real number system
- Metric spaces and point set topology
- Sequences and subsequences
- Series, including power series
- Continuity
- Differentiable functions
- Riemann integrals

Math 113

113 is an introductory course in the topic of complex analysis, a field which generalizes notions of multivariable calculus (differentiation, line integrals) to the complex plane. The structure provided by the multiplication of complex numbers has some beautiful and amazing consequences, which gives rise to substantial differences between the study of complex differentiable functions and smooth functions. This subject has important applications to many fields of mathematics, including number theory, algebraic geometry, differential geometry and PDE, among others.

Topics Covered

- The complex plane
- Complex differentiation and the Cauchy-Riemann equations
- Integration along paths and Cauchy's theorem
- Cauchy's integral formula and power series representation of holomorphic functions
- Conformal mapping and harmonic functions
- Singularities, the Laurent series, and meromorphic functions
- The residue theorem
- Applications of contour integration to real integrals

Math 114

This is a second course in real analysis; it provides a rigorous introduction to measurable functions, Lebesgue integration, Banach spaces and duality.

Topics Covered

The topics covered vary slightly depending on the instructor, but here is a possible list.

- Measure and Integration
  - Real numbers; open sets; Borel sets.
  - Measurable functions. Littlewood's 3 principles
  - Lebesgue integration
  - Monotonicity, bounded variation, absolute continuity
  - Differentiable and convex functions
- Banach spaces
  - The classical Banach spaces
  - Hilbert spaces and Fourier analysis
  - Topological spaces; $C(X)$
  - Compactness; Arzela-Ascoli
- Functional Analysis
  - Hahn-Banach theorem
  - Baire category
  - Open mapping theorem, closed graph theorem, uniform boundedness principle
  - Weak topologies, Alaoglu's theorem.
  - Measures as the dual of $C(X)$

Math 115

This course covers some of the most important analytical tools widely used in engineering and physics. It includes complex contour integration, differential equations, and Fourier analysis.

Topics Covered

- Basic complex analysis
- Ordinary differential equations
- Laplace transform
- Fourier series and Fourier transform
- Special functions
- Laplace equations
- Calculus of variations

Math 116

Math 116 is a course in real analysis and functional analysis (infinite-dimensional vector spaces) with the mathematics motivated by optimization problems. The basic strategy is to find approaches that can be visualized in two or three dimensions and to use them as the inspiration for theorems that apply even in the infinite-dimensional case. This makes it possible to solve problems where the solution is a countably infinite set of values or a function. Classical calculus of variations is a well-known example of this approach.

The emphasis in Math 116 is on mathematics, not on algorithms for solving optimization problems, but there are plenty of concrete examples.

Target Audiences:

- Applied Math concentrators who wish to fulfill the optimization requirement.
- Graduate students in Economics taking Economics 2010.
- Students doing a secondary field in Mathematical Sciences.
- Economics concentrators who are planning to go on to PhD programs or to work for firms that do sophisticated research.
- Math concentrators who began with Math 23, 25, or 55 and want to see some applications while meeting the real analysis requirement.

Prerequisites:

Math 23, 25, or 55 is fine. Sophomores have done very well.

If you took Math 21, you should also have taken another course where you did proofs. Obvious candidates are Math 101, Math 112, Math 121, Math 130, Math 152, and Math 154.

Topics Covered

- Review of vector spaces, convergence, limits, and continuity
- Hoelder and Minkowski inequalities; Lebesgue integrals
- Cauchy sequences, completeness, Banach spaces, separability
- Hilbert spaces, projection theorem
- Applications to optimal controll and portfolio design
- Linear functionals, normed dual spaces, representation theorems
- Hahn-Banach theorem and applications
- Calculus of variations with appliactions
- Concave and convex functionals
- Fenchel duality with applications

Math 117

Math 117 teaches probability in the context of the theory of option pricing. It is appropriate for students with a strong interest in finance or mathematical economics. The only prerequisite is solid single-variable calculus along with a taste for proofs.

Topics Covered

1. Exponentials and compound interest
2. Fair games, hedging, and arbitrage
3. Sets and sigma fields
4. Partitions and filtrations
5. Borel field; measurable functions
6. Continuity and convergence; probability spaces
7. Binomial model for call options; Poisson distribution
8. Independence; random variables
9. Expectation of simple random variables
10. Lebesgue integrals for expectation
11. Positive random variables
12. Monotone convergence theorem
13. Dominated convergence
14. Expectation and variance
15. Riemann integrals; central limit theorem
16. Convex functions; product measures
17. Call options and conditional expectation
18. Conditional expectation and hedging
19. Discrete martingales
20. Martingale convergence; continuous martingales
21. Share prices as random variables
22. Call options and Black-Scholes
23. Black-Scholes partial differential equation
24. Black-Scholes and perfect hedgng

Math 118r

This course is an introduction to theory and applications of dynamical systems, both continuous-time and discrete-time. The course integrates rigorous theory with numerous applications of genuine scientific interest.

Topics Covered

Topics covered include:

- Linear constant-coefficient systems of ordinary differential equations (ODEs);
- nonlinear ODEs: existence and uniqueness, local theory;
- nonlinear ODEs: global existence theory, nullclines, trapping regions, dependence upon parameters and initial conditions;
- flows on manifolds (mainly cylinder and torus);
- behavior near equilibrium, local stability, Lyapunov functions, stable and unstable manifolds;
- limit cycles, conditions for existence and non-existence, approximation using perturbation theory;
- bifurcation from equilibrium, Lyapunov-Schmidt reduction, steady-state and Hopf bifurcation theorems;
- global bifurcations, different routes to chaos; and
- bifurcations leading to chaos in discrete-time dynamical systems, Lyapunov exponents, symbolic dynamics.

Math 121

Math 121 is a proof-based linear algebra course, to be taken after a first linear algebra course such as Math 21b or 19b. No background with proofs is assumed, so Math 121 can also serve as an introduction to proofs.

The focus of Math 121 is to develop students' skill in writing proofs and to rigorously justify the linear algebra results that students have seen in their first course. In addition, students will see how these results can be applied to other parts of mathematics as well as to computer science, machine learning, and physics.

Topics Covered

1. Vector spaces, subspaces, and bases
2. Linear transformations and matrices
3. Elementary matrix operations and solving systems of linear equations
4. Eigenvalues, eigenvectors, and diagonalization
5. Determinants
6. Jordan canonical form

Math 152

Mathematics 152 provides a survey of topics which, in contrast to those studied in calculus courses, do not involve the infinite or the very small. In some cases (e.g., groups) the finite case is the traditional way to introduce the topic, but in other cases (e.g., topology and geometry), the finite case is usually ignored.

Many of the topics are relevant to theoretical computer science. With their knowledge of these topics plus the skill with proofs that they will acquire, students can go into Computer Science 121 or 124 without taking CS 20. Here are four novel features of the course:

- Students begin by constructing paper models of the dodecahedron and other Platonic solids, which then turn out to be closely related to permutations, groups, finite geometry, and linear algebra over finite fields.
- The course is taught in a seminar format, which is why it has an enrollment limit of 16. Everyone in the class will do a presentation at least once per week.
- Category theory, which seeks to unify almost every branch of mathematics, will be used as a organizational device. No background will be assumed beyond linear algebra and the concept of function.
- For almost half of the homework, an alternative is to write computer programs (in any language) tht implement the mathematical ideas presented in the course.

For students who have taken only Math 21a and 21b, Math 152 is a good way to learn proof strategies. For students from Math 23, the proof strategies will be familiar but the subject matter will be almost completely new.

Topics Covered

- Rotations of polygons and Platonic solids
- Permutations and permutation groups
- Axioms and theorems of group theory
- Rings and finite fields
- Finite geometry
- Finite groups of linear transformations
- Group homomorphisms and isomorphisms
- Elementary graph theory
- Graphical representations of finite groups
- Logic, done using finite fields
- Finite topology
- The category of abstract sets
- Groups, graphs, and matrices as categories

Math 153

Introduces basic concepts of mathematical biology and evolutionary dynamics: evolution of genomes, quasi-species, finite and infinite population dynamics, chaos, game dynamics, evolution of cooperation and language, spatial models, evolutionary graph theory, infection dynamics, somatic evolution of cancer.

The class includes a number of guest lecturers to expose students to current topics in the field.

Topics Covered

- Evolution of Cooperation
- HIV Infection
- What Evolution Is
- Fitness Landscapes and Sequence Spaces
- Evolutionary Games
- Prisoners of the Dilemma
- Finite Populations
- Games in Finite Populations
- Evolutionary Graph Theory
- Evolutionary Dynamics of Cancer
- Cancer Treatment
- PreLife
- Virus Dynamics
- Evolution of Language
- Experimental Game Theory
- Evolution of Eusociality

Math 154

This course is a rigorous advanced introduction to probability without measure theory. It provides a strong foundation in probability for students interested in mathematics, physics, or computer science.

The topics covered vary depending on the instructor, but here is a possible list.

Topics Covered

- Random variables (discrete, continuous, multi-dimensional distributions, etc.)
- Conditional probability
- Characteristic and generating functions
- The law of large numbers
- Central limit theorem
- Random walks
- Markov chains
- Martingales
- High-dimensional and geometric probability
- Entropy

Math 157

An interactive introduction to problem solving with an emphasis on subjects with comprehensive applications. Each class will be focused around a group of questions with a common topic.

Due to the interdisciplinary nature of the material, the course is accessible to students interested in computer science, statistics, and logic. Taking this course is also an ideal way to prepare for a variety of quantitative interviews. Students planning to pursue careers in technology, finance, and engineering should find the material beneficial when approaching the job market.

Topics Covered

Topics vary based on student interest, but here are some possibilities:

- Logic and information
  - Symmetry
  - Recursive reasoning
  - Tiling and packing
  - The pigeonhole principle
- Number theory
  - Prime numbers and prime factorization
  - Divisibility and remainders
  - The Fibonacci sequence
  - Generating functions
- Probability
  - Basic probability
  - Conditional probability
  - Geometric probability
  - Combinatorial probability
  - Random walks
- Strategies and decision making
  - Betting and investment strategies
  - Static games with complete information
  - Dynamic games with complete information
  - Bayesian games
  - Nash's existence theorem
- Algorithms and programming
  - Bitwise operations
  - Searching

- Hashing
- Sorting
- Dynamic programming
- Trees

Math 243

In this seminar-style course, students are exposed to current research areas in the field of evolutionary dynamics. Students are required to attend each class and conduct a project on an approved topic related to the course. This project should be original research supervised by a member of the Program for Evolutionary Dynamics (PED). As an alternative, students are permitted to conduct an original exposition on a body of existing research. Students will be graded according to participation in discussions and on their final projects.

Topics Covered

Topics may include:

- Cooperation
- Games in structured populations
- Evolutionary dynamics of diseases
- Luria and Delbrück distributions