PRESENTATION II TOPICS

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These topics are only some suggestions: please see the webpage for a further list of possible topics, and make sure to approve them with me beforehand. Also, any of the topics from Arnol’d’s or Frankel’s books would make good presentation topics. However, I would like there to be at least one student presentation on each of these topics.

Kepler’s Problem Revisited: We can use the geometric techniques studied in this class to give a very sophisticated solution to Kepler’s problem. Explain how Kepler’s problem in three dimensions has a hidden $SO(4)$ symmetry and deduce the existence of an additional conserved quantity, the Laplace-Lenz vector. Calculate the Poisson brackets and use Hamiltonian reduction to completely determine the motion. Reference: Guillemin and Sternberg, *Variations on a Theme by Kepler*, Chapter 1.

Geometric Quantization: Geometric quantization studies the question of how to produce a quantum-mechanical Hilbert space from any symplectic manifold. Explain how to set up the problem and give some simple examples of prequantization. Explain polarization and why Lagrangian submanifolds should correspond to quantum states. Reference: Blau, *Symplectic Geometry and Geometric Quantization*, Chapters 3 and 4.

Symplectic Toric Manifolds: Symplectic toric manifolds are some of the most important examples of symplectic manifolds, since they have a very concrete combinatorial calculus. Define symplectic toric manifolds and explain Delzant’s theorem relating symplectic toric manifolds to convex polytopes. Give some simple examples and explain the connection to symplectic reduction and the Arnol’d-Liouville theorem. Reference: da Silva, *Lectures on Symplectic Geometry*, Chapter XI.

Contact Geometry: Contact geometry is the odd-dimensional cousin of symplectic geometry, and is a very important modern field of geometry in its own right. Define a contact manifold and a contact form and give some simple examples. Explain how to obtain a contact manifold from an energy hypersurface in a symplectic manifold, and the role of the Reeb vector field. Reference: Arnol’d, Appendix 4;

The KAM Theorem and Applications: The Kolmogorov-Arnol’d-Moser Theorem is one of the most important results in the theory of Hamiltonian dynamical systems. Give an explanation of the theorem and its relation to the Arnol’d-Liouville theorem; give some applications to celestial mechanics and the stability of the solar system. Reference: Arnol’d, Section 51 and Appendix 8.

The Hamilton-Jacobi Method: The Hamilton-Jacobi equation gives a formulation of Hamiltonian mechanics that makes obvious its parallels with optics and quantum mechanics. Derive the Hamilton-Jacobi equation and give a simple example. Then explain how it arises as a short-wavelength approximation of the Schrödinger equation. Explain the connection to generating functions for canonical transformations. Reference: Arnol’d, Chapter 9.
Kähler Manifolds: Kähler manifolds are a class of symplectic manifolds that have incredibly rich and deep geometric properties, which makes them important in high-energy physics. Define a Kähler manifold, and explain why all smooth algebraic varieties give examples. Then explain the Hodge decomposition for a Kähler manifold and some consequences. Reference: Huybrechts, Complex Geometry, Chapter 3; or da Silva, Chapter 16-17.

Rigid Body Mechanics: Rigid body mechanics is another classical topic from mechanics that can be easily treated from a mathematically sophisticated perspective. Explain how to formulate rigid body mechanics in terms of the Lie group $SO(3)$ and how to apply Hamiltonian reduction to simplify the problem. Describe the connections to representation theory and hydrodynamics. Introduce the Euler angles as coordinates, and explain the phenomena of precession and nutation for spinning topics by deriving the Euler equations. Reference: Arnol’d, Appendix 2 and Chapter 6.