Recall \{G-schemes\}

\[
\text{Mor}(X, Y) = \text{Shv}(X \times \mathcal{Y})_G
\]

\[
\text{Mor}(X, Z) \otimes \text{Mor}(Y, X) \rightarrow \text{End}(X)
\]

\[
\rightarrow \text{Mor}(Y, Z)
\]

fully faithful

Apply this to get derived Saltok
\[ \mathcal{H} = GF / G_0, \]

\[ D_G / (\mathcal{H}) \left( \mathcal{H} \right) \leftarrow D(G_0 \setminus GF / (I_0, I)) \]

\[ D_G / (I_{0,1}^+ I_{0,1}^-) \cup \text{perm} \]

\[ I_0 < I - \text{reducible of Whittaker} \]

\[ \gamma : I_0 \rightarrow G_0 \]

\[ I_0, + - \text{equivalent - Baby Whittaker} \]
Here $D(\mathfrak{G}^\mathfrak{f}/\mathfrak{I}_0, Y)^{\text{full}} \subset D(\mathfrak{G}^\mathfrak{f}/\mathfrak{I}_0, Y)$ generated by the image of the averaging functor from $D(\mathfrak{G}^\mathfrak{f}/\mathfrak{I}_0, Y)$. This behaves as sheaves on an ind-projective quotient.

Derived Satake

$D_{G(0)}(Y_2) \cong D\mathcal{C}h^G(\mathbb{Z} \times \mathbb{Z})$. 

1) $$\text{Thm: } D \left( \mathbb{GF}/I_{\mathbb{GF}}, + \right) \simeq D^b \text{Rep}(G)$$

"geom. = casselman + shalika". Freydel

Vilenen

Feynsma 2000 with $\mathbb{C}$ coefficients.

B & Mirkovic Riche Rider

with modular coeff.

$$\text{Rep} \left( \mathbb{GF}/I_{\mathbb{GF}}, + \right) \simeq D^b \text{Coh}^\ast \left( G^\ast \right)$$

2) (w. Riche, Rider in progress)
Recall Gaitsgory's central functor $\mathbb{Z} : \text{Rep}(G^\vee) \to D(\mathcal{F})$

Defined by nearby cycles carries a tensor automorphism (mediocrity) allowing to extend it to a functor from $\text{Coh} \ G^\vee$.

\[ F = V \otimes \Omega \]

Version: $\hat{\mathbb{Z}} : \text{Rep}(G^\vee) \to D(\Leftrightarrow ?)$
Hope: version of this works for $(\overline{\text{GF}} / I_0, \Phi)$ unip.

Not worked out.

Another approach

Consider

$$D\left(\overline{\text{GF}} / I\right) = \Phi$$

$I, I^- \subset G(0)$

Opposite

Have obvious functors

$$D\left(\overline{\text{GF}} / I_0, \Phi\right)$$

$$\cong$$

$A\nu_{un}$
Observations.

It's easy to describe the kernel of $A_{\text{wh}}$.

$\text{Im} (A_{\text{wh}}) = \{ \ell \mid \ell \in \mathbb{W} \}$.

$\text{Ker} (A_{\text{wh}}) = \langle \mathbb{W} \mid \mathbb{W} \in \text{min} \rangle$.

$\mathbb{W}$ - minimal in $A_{\text{wh}}$ 2-sided case.

$\text{Ker} (A_{\text{wh}})^{-1} = \mathbb{R}$.

$\mathbb{R}$ can be described using $\mathbb{E}$ & its properties.
Now $R$ is "almost equivalent" to $\text{Wh}$.

More precisely, Wh is naturally a category over $T^v/W$. (sat. th. supp at $\mathbf{1}$).

\[
\text{Prop.} \quad R \cong \text{Wh} \otimes \text{Ch}(T^v \times T^v) \text{Col}(T^v/W) T^v/W \\
\text{W x W acts on both sides.}
\]

Then $\text{Wh} \cong \text{Col}(G^- \times T^v \times T^v) T^v/W T^v/W$.

$W^2$ equivalently $\Rightarrow \text{Wh} = \text{Col}_{G^-}(G^-)_{\mathbf{1}}$. 
Prep & formalism of last lecture
work in progress with T. Deshpande.

// modular version of a result of Ginzburg
// (for D-modules)

we show

\[ D(\mathcal{U}_G(U_Y)) \cong D(\mathcal{I}_W^{-}) \]

G-reductive group explicit

\[ D(\mathcal{U}_G(U_Y)) \cong D\text{Coh}_T(U_Y^{-}) \cong D\text{Coh}_T(W) \]
\( \text{D} (X / u, y) \otimes \text{D}(u, y, G / u) \text{D}(u, y, G / u, y) \uparrow \text{mp} \)

\( \text{D}(X / u -) \)

\( \otimes \text{Ch}(\tilde{T}) \)

\( \text{Ch}(\tilde{T} / w) \)

lands in the right orthogonal to kernel of the averaging functor

1 and in an equality with that kernel at least.
Proof of the Thin.
Recall \( D (E) \to D(G/H, Y) \)

The kernel is generated by \( L w, w \neq 1 \).

This orthogonal is

\[ \text{projective pre-cover of } \mathbb{Q} \]

(in the monad of)

setting \( R \) to volbe of \( \mathbb{Q} \).

Our \( R \) is generated by
$R = \langle Z(V) + \Xi \rangle$.

1) in modular setting can restrict

So $V = T^+ \mathrel{\text{tilting}}\mathrel{\text{in Rep}}(\mathcal{Z})$

Lemma 1 $\Xi \mathrel{\text{tilting}}$

$\Xi \mathrel{\text{tilting}}$

$\Xi \mathrel{\text{he free monodromy tilting}}$

$\text{Ext}^0(\quad) = 0$

$\text{Hom}(\quad) = \text{Hom}(T \otimes O, T \otimes O)$
- proved by analyzing the quotient

\[ D(\text{GF}(\ell)) / \mathbb{Z} \]

\[ w \neq 1 \]

\[ \rightarrow \text{Col} / \mathbb{Z}_{\text{neg}} \]

in strongly

on Kostant slice