Given a hypersurface \( H^{n-1} \) over a nonarchimedean field \( K \), degenerating family of complex hypersurfaces near tropical limit embedded in alg. toric variety, or abelian variety \( V^n \) 
\( \rightarrow \) mirror LG-model \((Y,W)\) where \( Y \) is a CY \((n+1)\)-fold, \( W \in \Omega(Y) \) sympotential.
(\(\text{depends on choice of embedding}\))

\[ f(x) = \sum_{x \in \mathbb{R}^{n+1}} c(x) x^t \text{ Laurent polynomial (} t \to 0) \]
\[ \Rightarrow \text{tropicalization } \tilde{f}(\tilde{x}) = \max_{x \in \tilde{X}} \left( \langle \tilde{x}, \tilde{f} \rangle - f(x) \right) \]

\(*\) let \((Y,W)\) be toric CY with moment polytope \( \Delta_Y = \{ \tilde{f}(\tilde{x}) \} \subset \mathbb{R}^{n+1} \)

\[ W_0 = -e_0 \]

toxic moment which vanishes to order 1 on each toxic divisor of \( Y \)
\( \Rightarrow (Y,W_0) \text{ is mirror to } H \subset (\mathbb{C}^*)^n. \) (eg. SYZ mirror to a LG-model w/Morse-Bott sign along \( H \))

\[ \begin{align*}
\mathbb{C}_2 \setminus \mathbb{C}_3 & \quad \Rightarrow \begin{cases}
\frac{\mathbb{C}_2}{0}, \quad \mathbb{C}_3, \quad W_0 = -z_1z_2z_3,
\gamma = \max(0, \tilde{f}, \tilde{f}_2) 
\end{cases}
\end{align*} \]

more generally, \( \mathbb{T}_{n-1} = \{ z_1 + \cdots + z_n + 1 = 0 \} \setminus (\mathbb{C}^*)^n \)

\[ \begin{align*}
z_1 + z_2 + t/z_1z_2 + 1 = 0 & \quad \Rightarrow \begin{cases}
\frac{\mathbb{C}_2}{0}, \quad \mathbb{C}_3, \quad W_0 = -u^2z_1z_2z_3,
\end{cases}
\end{align*} \]

\[ \begin{align*}
(2) & \quad \Rightarrow \begin{cases}
\frac{\mathbb{C}_2}{0}, \quad \mathbb{C}_3, \quad W_0 = -u^2z_1z_2z_3,
\end{cases}
\end{align*} \]

\[ \begin{align*}
\begin{cases}
\mathbb{C}_2 \setminus \mathbb{C}_3 \quad \Rightarrow \begin{cases}
\frac{\mathbb{C}_2}{0}, \quad \mathbb{C}_3, \quad W_0 = -u^2z_1z_2z_3,
\end{cases}
\end{cases}
\end{align*} \]

Remark 1: \( \bullet \) the smooth fiber of \( W_0 \) is \((\mathbb{C}^*)^n \)
\( \bullet \) the sing. fiber \( W_0^{-1}(0) = U \) toxic strata in \( Y \)
\( \bullet \) critical locus = \( U \) cliched 2 toxic strata - for curves with max. degeneration ("tropically smooth") this is a trivalent configuration of \( C \)'s and \( \mathbb{P}^1 \)'s

Remark 2: in simple examples, \( Y = \) line bundle over some toric var., \( W_0 = U \cdot e \) \( e \) \( \mathbb{P}^1 \) base by Orlov's Knörrer periodicity \( \text{B}^* (\mathbb{C}^*, W_0) = \text{B}^* \text{Coh}(e^{*} \mathcal{O}) \) (usually singular) \((n+1)\)-st \( \mathbb{P}^1 \) mirror

eg. \( (1) \) points \( \{ z_1z_2 = 0 \} \leftrightarrow \{ z_1z_2 = 0 \} \subset \mathbb{C}^2 \) (similarly \( \mathbb{T}_n \leftrightarrow \{ \mathbb{T} \} \subset \mathbb{C}^n \))

eg. \( (2) \) points \( \{ z_1z_2 = 0 \} \) \( \mathbb{C}^2 \)

\[ \begin{cases}
\mathbb{C}_2 \setminus \mathbb{C}_3 \quad \Rightarrow \begin{cases}
\frac{\mathbb{C}_2}{0}, \quad \mathbb{C}_3, \quad W_0 = -u^2z_1z_2z_3,
\end{cases}
\end{cases} \]

For higher genus, such minima are strongly non-reduced - don't consider.
For $H \subset V^n$ toric var., def by $f(x) = \sum_{x \in \mathcal{A}} t(x_i) x_i^{x_i} \in H^0(V, \mathcal{L})$
(Curv($A$) = Nekr($x_1$))

mirror is $(y^{|x+1|}, v^0 + v^1)$
extra terms of superpot., one per ray of $E_v$
$\sum_{\nu \in \text{rays of } E_v} t(\nu) z(\nu, \lambda(\nu))$
$\lambda(\nu) \in \mathbb{R}, \text{ PL function}$
defining $x = O(H)$

developed by Tam$f$

Ex. (1) \{x_0 + x_1 + x_2 = 0\} $\subset \mathbb{P}^2$ $\leftrightarrow (x^3, -z_1 z_2 z_3 + T(z_1 + z_2 + z_3))$

Ex. (2) $\bigcirc \subset V$ $\leftrightarrow (y^{|\text{deg}(3)|}, -u z_0 z_1 z_2 + T u (z_0^3 + z_1^3 + z_2^3))$

(NB: Not in toric. Both along smooth cubic elliptic curve)

Ex. (3) genus $g$ curve $< \text{toric Del Pezzo} < \leftrightarrow \text{cut}(w_0 + w_1) = \text{bivert configuration of}$
$\text{deg}(g - 3)$ IP's meeting in $(2g - 2)$ nodes

eg: genus 2 $\bigcirc \leftrightarrow \bigcirc = \text{cut}(w_0 + w_1)$

* These contractions also work in abelian varieties (see eg. C. Cannizzo's thesis)
& for complete intersections ($H^{n-k} \subset V^n < \leftrightarrow y^{n+k}$, $w_0$ has $k$ terms).

**HMS related:**
(A) $W(H)$ (wrapped Fukaya cat. of $H$) $\leftrightarrow \text{Dg}(Y, W)$ (or $\text{D}^b \text{Coh}(\mathcal{O}$'s) via Orlov)
well studied: AAEKO, H. Lee, Nadler, Gammage-Shende, Lekili-Polishchuk, ...)
for reasons in $\mathcal{O}^2$ $\text{toward higher dim, but C^* singularity is sheaves. GPS no same thing!}$

(B) $\text{D}^b \text{Coh}(H)$ to $F(Y, W)$:

1) via Fukaya wrapped Fukaya cat (Abouzaid-A.)
2) via $F(\mathcal{O}'(0))$ when expect a non LG mirror via Knörrer periodicity (H. Jeffs)
3) by directly working on cut(W) (or cut(W), U) Bypass the need to embed H into V ??

our `focus`

- for mirrors of curves: Log. Pieri theory in bivert configurations of C's and IP's?

(A. Efimov-Katzarkov in progress)

Today, focus on (1)
The fibrewise wrapped Fukaya cat. of $(Y, w_0)$ or $(Y, w_0 + L_0)$ [Abouzaid-x.

(ad hoc construction for toric LG models - can extend using language of sectors ... A non-exact Y)

Using monomial admissibility, see also Hanlon's thesis

Objects: properly embedded Lagrangians $L \subset Y$ which are

- topologically unobstructed (don't bind below $b_{1,0}$)
- equipped with extra data (spin structure, grading, local system)
- monomially admissible;

$\Rightarrow$ for $|w_0| > 1$, $\arg(w_0) \leq \text{constant}$ (ie. $w_0 |_L \in \text{union of radial arcs}$

$\in (-\frac{\pi}{2}, \frac{\pi}{2})$)

$\Rightarrow$ recalling that fibres of $w_0$ are $\cong (\mathbb{C}^*)^n$, outside of a compact subset of

these, $\exists$ finite open cover $\{U_i\}$ and a collection of monomials $z^\nu$

[exactly those in $w_0$ if $V$ compact, else choose a toric compactification $\overline{V}$

using $\text{Neut}(f) = \text{Conv}(A)$!]

$\Rightarrow$ $\arg(z^\nu)|_{L \cap U_i}$ bounded constant (taking prescribed values, eg 0).

Ham. perturbations: $L$ admissible $\Rightarrow$ flow $L^t$ (Hamiltonian isotopic to $L$; admissible)

the flow increases the values of $\arg(w_0)$ and $\arg(z^\nu)$ at $\infty$

- for $w_0$ and for $z^\nu$ which appears in $w_0$, $\nu$ in bounded interval - NO WRAPPING
- for other monomials (in $w_0$, from compactification but not in $w_0$), $\nu$ to $\infty$ (WRAP)

Then define $\text{hom}(L_0, L_1) = \lim_{t \to \infty} C\text{F}^\infty(L_0^t, L_1)$ under natural continuation maps.

* The fibrewise partial wrapping Hamiltonian is essentially the one which appears in Hanlon's thesis as

monodromy for families of toric mirrors / induced functor on $\text{F}((\mathbb{C})^n, L_0) \cong \text{D}(\overline{V})$

is, under HOS, mirror to $-\otimes \mathcal{O}(D), D = \overline{V} - V$ compactification divisors.

So the direct limit amounts, fibrewise, to the localization which restricts from $\text{D}(\overline{V})$ to $\text{D}(V)$

* Ex: $(\mathbb{C}^3, w_0 = -z_1 z_2 z_3)$ (mirror of $\mathbb{A}_3$), $L_0 = \text{parallel-transport} \ l_0 = (R^t)^2 \subset (\mathbb{C}^*)^3 - w_0^{-1}(-1)$

along $U$-shaped arc.

$w_0^{-1}(-1) = (\mathbb{C}^*)^2$

$l_0 = (R^t)^2$

$\Rightarrow$ images of $l_0$ under monodromy + wrapping fun. $\text{hom}$

$\arg(z_i) = \text{f}(\log |z_i|)$. 

$\Rightarrow$ $\arg(z_i) = 0$ wherever $|z_i| > \min |k_j|$. 

Wrapping

$t \ (\text{here } t \to \infty)$.
\[ \text{Area} = \int_{0}^{1} \int_{0}^{1} (1+x+y) \, dx \, dy \]

\[ = \left[ x \right]_{0}^{1} \int_{0}^{1} (1+x+y) \, dy \]

\[ = \left[ y + \frac{y^2}{2} + y^2 \right]_{0}^{1} \]

\[ = 1 + \frac{1}{2} + 1 = \frac{5}{2} \]