Material covered: category language; paths and homotopy (§51).

1. (5 points) Let $X$ be a topological space, and consider the equivalence relation on $X$ defined by $x \sim y$ if there exists a path in $X$ from $x$ to $y$. The equivalence classes are called path components of $X$. Define $\pi_0(X)$ to be the set of path components of $X$.

(a) If $f : A \to Y$ is continuous and $A$ is path-connected, show that $f(A)$ is path-connected and thus contained in a single path component of $Y$.

(b) Show that if $f : X \to Y$ is a continuous function, there is an induced map of sets $\pi_0(f) : \pi_0(X) \to \pi_0(Y)$.

(c) Show that $\pi_0$ is a functor from the category of topological spaces (with continuous functions) to the category of sets.

2. (4 points) Munkres exercise 51.2.

3. (6 points) Munkres exercise 51.3.